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The Coordinated Movements of a Learning Assemblage: Secondary School Students Exploring Wii Graphing Technology

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Abstract This chapter uses assemblage theory to investigate how students engage with graphing technology to explore mathematical relationships. We use the term 'learning assemblage' to describe provisional dynamic physical arrangements involving humans and other bodies moving together and learning together. Emphasis on dynamic coordinated movements allows us to study how mathematics learning occurs in complex interaction with technology. We tap into the rich concept of 'sympathy' to understand the way that students develop a feeling for these coordinated movements as they participate collaboratively in mathematical investigations. Through sympathetic movements, a learning assemblage sustains a kind of *affective agreement* amongst the various bodies that participate. We show how assemblage theory helps us rethink the role of affect in technology tool use. This chapter sheds light on innovative ways of theorizing the role of Wii graphing technology in mathematical practice.

1 Introduction

In this chapter we explore how a sense of coordinated movement is entailed when students use Wii graph technology to explore mathematical relationships. We use assemblage theory and its *emphasis on relations between movements* in order to understand how these students are doing mathematics. The concept of *assemblage* has been taken up and used extensively in various new materialisms and new

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empiricisms in the social sciences (Bennett, 2010; de Freitas, 2012; Fox, 2011; Mazzei, 2013). Much of this work follows assemblage theory as articulated by Gilles Deleuze and Feliz Guattari. According to this approach, assemblages are the fundamental “real unit” of study. Deleuze and Parnet (2007), for instance, claim that “the minimum real unit is not the word, the idea, the concept or the signifier, but the *assemblage*” (p. 51). In the inclusive materialist perspective of de Freitas and Sinclair (2014), the notion of assemblage is offered to de-essentialise the body and rethink its contours in mathematical activity, so that the *potentiality* of the body is stressed. Our focus in this chapter is on how human bodies collaborate and assemble with technology when exploring mathematical ideas. In another contribution in the book, Sinclair and Coles (Chap. Returning to Ordinality in Early Number Sense: Neurological, Technological and Pedagogical Considerations) draw on inclusive materialism to speak similarly of the role of the material environment in mathematics teaching and learning, shifting attention to the assembling of the human body with the concept of number. This chapter attempts to address some of the concerns raised about the concept of assemblage, particularly the concern that it is used all too often to simply name a *set* of individuals in a relationship (Buchanan, 2015). Indeed, we believe that the power of assemblage theory lies in the way it emphasizes how bodies are provisional relationships between *moving* parts, and that this *coordinated* movement involves an *affective bond* between components. In other words, a body is assembled through the dynamic force of affect, and not simply through mechanistic coordination of a set of components.

We use the term *learning assemblage* to describe provisional dynamic physical arrangements involving humans and other bodies *moving together* and *learning together*. Drawing on assemblage theory, we argue that perception occurs across these provisional arrangements and not in one central processing location (like the brain). This allows us to better study the way that acts of perception involve collaborative movement and activity. To “perceive” is actually to *assemble with* a relational environment in such a way as to fold back into it.

Such an approach highlights the concept of *proprioception*, which was originally defined as “sense of locomotion” and has evolved into the idea of “muscle sense” and a sense of one’s own body’s configuration. Proprioception is, by definition, a relational property of any assemblage. For instance, proprioception explains how we can move rapidly and without reflection in order to grasp a falling cup from a table. As one moves, the “proprioceptive potentialities” (p. 38) of the body are continuously reconfigured, as are the relative locations of objects in the foreground and background. This insight resonates with phenomenological approaches to perception, whereby *corporeal space* is “lived spatiality, oriented to a situation wherein the lived/living/lively body embarks on an architectural dance that actively spatializes (and temporalizes) through its movements, activities, and gestures.” (Coole, 2010, p. 102). Proprioception is part of a larger concept called *kinesthesia*, which refers to the ability of the human body to feel its own movement and states, and thereby contributes to the sense that ‘oneself’ is the source of such action (Streeck, 2013). Sheets-Johnstone (2012) argues “not only is our perception of the world everywhere and always animated, but our movement is everywhere and always kinesthetically informed" (p. 113). These two ideas—proprioception and kinesthesia—are pivotal to our understanding of learning assemblages.

We use the concept of 'sympathy' to study the affective nature of coordinated movement in a learning assemblage. Although sympathy has various common sense meanings, we draw from the work of Gilles Deleuze who reclaimed the concept and tapped into its pre-Kantian meaning. For Deleuze, sympathy is a matter of independently moving bodies *moving together*, and involves the power of a body to affect and be affected (Deleuze & Parnet, 2007, p. 53). Sympathy thus takes on a pivotal role in understanding how affect is entailed in any learning assemblage. Following Deleuze and Parnet (2007), this ancient notion of sympathy, a term that comes from ancient Greek (sumpátheia) combines the meaning of "come together" and "pathos" and helps us understand how different bodies feel each other's movements. The notion of sympathy came to be used in diverse ways, but here we are interested in how it refers to a kind of agreement between bodies whereby they are mutually *affected* by each other through a coordinated movement. In our case, we want to study the way that affect plays a part in coordinated movements of different students insofar as they participate in a kind of collaborative and compassionate movement. Thus the learning assemblage is achieved insofar as affect sustains an agreement amongst the various bodies that join in. It is crucial that the term "agreement" not be interpreted as a judgement of rightness, but is rather a way of describing how bodies move together: "There is no judgment in sympathy, but agreements of convenience between bodies of all kinds" (Deleuze & Parnet, 2007, p. 52).

We next discuss a teaching experiment using Wii technology. We first present the context of research and the technology entailed, then the teaching experiment and finally a discussion of the data in terms of assemblage theory, drawing on the work of Manuel DeLanda (2006, 2011) who has further developed the ideas of Deleuze and Guattari.

2 The Context and Wii Technology

The experiment we present here took place in a secondary school in Northern Italy, as part of a wider study carried out during regular mathematics lessons. The study involved a class of grade 9 students participating in activities aimed at introducing the concept of function through a graphical approach using digital technology. The class was heterogeneously composed of 30 students (20 males and 10 females) from Torino and surroundings. The study lasted for 4 months and consisted of 9 meetings of 2 h in the period December 2014–March 2015. Two researchers (the second and the third author) designed and orchestrated the activities, while the teacher collaborated as an active observer within the classroom. The instructional methodology that was adopted offered diverse perspectives on the students' experience: collective discussions, group work, and individual work, often by means of written worksheets. The meetings took place in a laboratory room, which is used in the school as a laboratory for mathematical practice.

The activities were conceived so that the researchers could focus on kinaesthetic and proprioceptive experiences with tools that mobilized mathematical concepts related to functions. The approach draws on mathematics education research literature, which highlights that kinaesthesia and proprioception are part of mathematical understanding (see Nemirovsky, 2003; de Freitas, 2012, 2014; Ferrara & Ferrari, 2015; Roth, 2015). The teaching experiment focused on the spatio-temporal relationships that allow students to capture and describe motion phenomena, so it is greatly relevant for the study of mathematical functions, in part due to the historical roots of this particular area of mathematics, based as it was on the study of movement (e.g. Edwards, 1979). This research makes use of technology that is related to the game console Nintendo Wii because of the potential that it offers in terms of playing games through proprioception and kinaesthesia. The devices under consideration are the remote controllers (also called Wii Remotes, or Wiimotes) and the Balance Board of the Wii. The remote controllers are devices with which users can control and play games where real movement simulations are produced. The Wii balance board is usually used for games that depend on balance and body perception in space. So, bodily activity is crucial during activities performed with the Wii: the movement of the controller in the hand, the board under the feet, susceptible to all the variations in the player's balance, the eyes gazing at the feedback on the screen. The bodily actions required are kinaesthetic activities that deeply involve the proprioceptive capacities of the person who is participating. In a similar manner, Baccaglini-Frank and Robotti's contribution (Chap. Using Digital Environments to Address Students' Mathematical Learning Difficulties) discusses proprioceptive and kinaesthetic interactions with specific software as ways of accessing mathematical thinking by learners with disabilities.

The very first challenging step in this research project was to understand *how* to use the Wii devices in suitable pedagogical ways. Growing attention to *gamification* and *serious games* paradigms for education was seen as a way to tap student willingness to engage with the technology, which led to considerations of how to use the Wii as a resource for mathematics thinking and learning within a game context. Indeed, for many students such technology is already associated with game experiences, where players use the Wii technology to move through and solve problems within a virtual environment. Moreover, research suggests that affect might play a huge role in these kinds of game experiences. We were able to bring this technology to bear on pedagogical concerns through the use of two software applications, *WiiGraph* and *DarwiinRemote*, respectively working with two Wiimotes and one Balance Board. The first application has been developed with didactic goals by a group of researchers in mathematics education at the Centre of Research in Mathematics and Science Education of San Diego State University: Ricardo Nemirovsky and his colleagues (Nemirovsky, Bryant, & Meloney, 2012). The second software is freely available online. WiiGraph opened a wide range of opportunities to work with Cartesian graphs generated using Wii remotes. As the player moves her remote, the graph is depicted in real time on a single plane and captures instant by instant the movement of the corresponding controller. The graph on the screen documents the distance of the remote from an origin point, given by a sensor bar, which is positioned in the interactive space. Two players can play at the same time, and two different graphs can be shown on the screen.

Using a different modality, WiiGraph also allows working with a single graph which *assembles* the movements of the two remotes. This entails processing and integrating the two different movements in terms of one movement—that being the one graph that is collaboratively produced. These kinds of graphs lend themselves to two-person collaborative tasks involving two spatial variables, and can include activities of creating a rectangle or circle or some other figure, where one player controls the x-coordinate and another controls the y-coordinate. In a similar way, when connected to a Balance Board with a person standing on it, DarwiinRemote furnishes a dotted line on the screen. This line documents instant by instant the position of the person's centre of gravity, depicting the dotted graph of its projection on the horizontal plane (the plane of the board). This graph captures the horizontal motion trajectory of the centre of gravity. Beyond the original aim, the combined use of both software tools in the classroom allowed for rich explorations of the relations between motion laws and corresponding planar motion trajectories in the context of modelling motion. Even though we do not expand discussion about them here, design principles also play a role in our research in terms of novelty as for the studies presented in this book especially by Kynigos (Chap. Innovations Through Institutionalized Infrastructures: The Case of Dimitris, His Students and Constructionist Mathematics), Maschietto and Soury-Lavergne (Chap. The Duo "Pascaline and e-Pascaline": An Example of Using Material and Digital Artefacts at Primary School), and Tabach and Slutsky (Chap. Studying the Practice of High School Mathematics Teachers in a Single Computer Setting).

This chapter centres on a specific teaching experiment concerning sinusoidal functions and their relationships with a circular trajectory. In the experiment, the students made use of the first software, WiiGraph, to work with two types of graphs: *Line* graphs and *Versus* graphs.

3 WiiGraph Software: *Line* and *Versus* Graphs

WiiGraph is an interactive software application that takes advantage of the Wiimotes' multiple features to detect and graphically display the location of two users as they move along life-size number lines. In our experiment, an interactive whiteboard was also present in the classroom for visual experiences with the graphs projected by the computer screen, as well as a wide interaction space in the middle of the room to enhance students' opportunities for embodied and kinaesthetic experiences with the controllers.

A graphing session with WiiGraph commences when each user holds the controller pointed toward the sensor bar, so that a diffuse circle, matching a specific colour for each Wiimote, appears in the graph area. The diffuse circle is an index of the fact that the bar is capturing the distance of the controller at that moment—the circle indicates that the sensing technology and the software are coordinated. Once

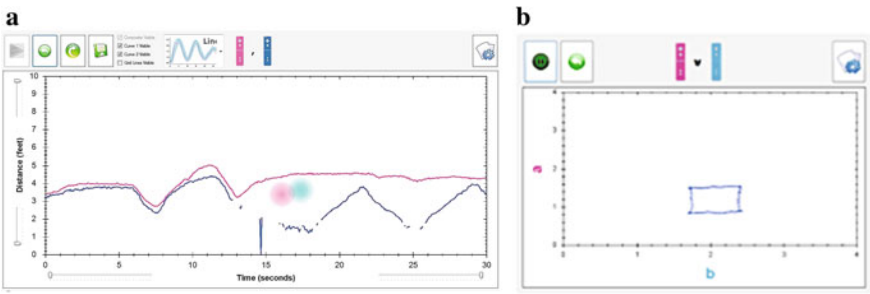

Fig. 1 **a** Graphs with *Line* modality. **b** Graphs with *Versus* modality

each circle is visible, WiiGraph can produce real time graphs with lines of the corresponding colour. Students can individually and collaboratively explore and work with several graph types, challenges, and composite operations, including shape tracing, maze traversal, and ratio resolution.

The graphs are configured in the graph area according to selected graph type, operations, and parameters like ranges, time periods and targets. Visibility controls can also be toggled during or after the session to selectively hide and show particular characteristics.

Among the various visual experiences that WiiGraph provides to the students, two are the most interesting graphical types for our study: *Line*, on the one hand, and *Versus*, on the other. We briefly discuss the two kinds of graphs to understand their functioning and the dynamic modes of interactions they offer to the students.

The *Line Graph* type, without target and operation, allows for depicting two distance-time lines of the kind $a(t)$ and $b(t)$, which correspond to the two Wiimotes' movement in front of the sensor bar, where a and b are the positions of the controllers and give their distances from the sensor. The thin coloured graphs are shown on the same Cartesian plane in the fixed time interval and they correspond to individual users (Fig. 1a). If an operation is selected, for example the sum $a + b$, this type adds to the previous lines a new distance-time line, which is the result of the operation at any given instant, in this case $(a + b)(t)$.

The other type, which is the very focus of our study, is the *Versus Graph* type. *Versus* plots an ordered pair of the distances of each user over time (the creation of the ordered pair is implicit). Briefly speaking, the graph that appears on the screen as result of the movement of the two Wiimotes is in this case the line $a(b)$, which is composed of the pairs of the kind $(b(t), a(t))$, for each t of the interval under consideration. The graph is thus always a spatial graph, where the variable of time disappears from the axes. In this perspective, one of the most significant challenges offered by *Versus* involves, as already said above, the creation of plane shapes, like rectangles, diamonds and circles (see e.g. Fig. 1b). Interestingly, this modality offers the students the opportunity of working together to collaborate and coordinate with each other for reaching a common goal.

4 The Teaching Experiment

In the teaching experiment discussed in this chapter, the students worked with WiiGraph in an activity focused on diagrams produced by the movement of two Wiimotes. At this point of the study, the students had already worked with WiiGraph on various aspects of functions through other activities using the *Line* modality: for example, they had explored plane transformations of graphs, such as translation and dilation; operations on functions, like the sum of functions; and relations amongst families of functions, like parallel straight lines, etc. They had also used the technology to face challenges that required matching suitable movements with given graphs (which offered room to reason on the role of the independent variable). Each of these activities required the students to use and compare with each other the two space-time graphs of $a(t)$ and $b(t)$. In this case, no constraint about coordinating the movements of the two Wiimotes in the interaction space was, implicitly or explicitly, given by the task. Rather, each learner could do a movement in a totally independent way with respect to the other learner. We discuss here a completely new activity, following these ones, in which pairs of students were asked to use the *Versus* modality of WiiGraph to generate together a single 'spatial graph' of the kind $a(b)$. These spatial graphs were the rectangle, the rhombus, and the circle (see again Fig. 1b). The novelty of the activity resides, at this point, in the fact that the students had to discuss ways of combining and coordinating the movements of the two controllers in order to produce one of these planar graphs. Concerning the rectangle and the rhombus, the students' first explored trials with the controllers. As their discussions evolved, they talked about how the changing positions of the two controllers were connected and assembled in the plane figure that they were seeing on the screen, referring to these positions as horizontal and vertical components of movement. Next, the task aimed to make this idea of horizontal and vertical components explicit for the whole class, asking two students, in front of their mates, to imagine being these orthogonal components on an imaginary vertical plane in space. The second author stood in front of them and gestured in space the plane figure (rectangle, rhombus), by moving her hand, while the two students had to move their right hands miming simultaneously the movements of the two components (see Fig. 2a for the case of the circle). This was purely a gestural affair, without the Wiimotes, in order to first explore the kinds of movements entailed in this task. The task involves the two learners in commencing their movements together with the researcher, synchronizing so that the learners' hands are always (1) at the same distance above the floor and (2) at the same width from the wall, as the researcher's hand. In so doing, the researcher's movement dictates the timing of the students' hand movements and the way in which they have to be assembled so that the original figure is the combined effect of such movements.

Then, the teaching experiment had the students return to the use of WiiGraph for obtaining a specific plane figure, taking advantage of the previous experience. Unlike the orthogonal gestures in the previous miming experience, the WiiGraph

Fig. 2 **a** Circle and components' movements in the air. **b**, **c** Lucrezia and Barbara coordinating the Wiimotes' movements

technology requires the two movements of the x and y coordinates (b and a, respectively) be performed *in parallel*, adding another complex dimension to the task. The WiiGraph software demands that the two controllers are moved along parallel lines in front of the sensor bar, although the sensory data is then processed as orthogonal. The movements, which before were orthogonally driven by the researcher's hand movement, and therefore more appropriately linked to habitual ways of characterizing these two components of planar movement, now need to be assembled in a different way. Indeed, the linear movements of the two Wiimotes have to occur along parallel lines, even though their combined effect will produce a similar two dimensional planar figure on the screen. In addition, the students need to agree with each other, driven by the software's feedback, so that the movements occur in suitable timing with each other and at suitable but different speeds for the combined movement to achieve the specific figure.

Thus the task entails tapping into time or duration in challenging ways, combining individual heterogeneous rhythms to achieve a third definitive rhythm. This third rhythm is then expressed as the target shape—be it the rectangle or rhombus. This task points to the fundamental role of time or duration in theories of embodied mathematics. It also shows how the learning assemblage implicates a confluence or commingling (like an orchestra or river) of diverse speeds and movements.

In this chapter, we focus on an excerpt from the video data collected during this teaching experiment when the students turned to the task of how to produce a circle using *Versus* and two Wiimotes. Two students, Lucrezia and Barbara, came in front of the class and mimed the gestures of the second author (Fig. 2a). They tried to explore the hand movements that might be needed to generate an imaginary circular trajectory in the air. Then, Lucrezia and Barbara started moving the Wiimotes with the aim of producing a circle as a third movement on the screen. Figs. 2b and c show the two girls while they are moving the controllers trying to be synchronized both in rhythm and speed. Fig. 3a shows the new circular movement that they are able to obtain on the screen (after some trials).

In order to better grasp the ways in which these coordinated movements of these two girls relate to the mathematics of the software and the mathematics of the figures they are making, we describe here the specific quantitative relationships that

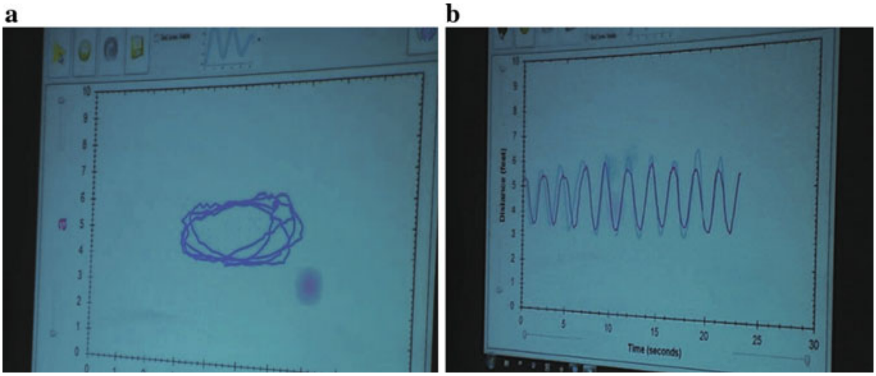

Fig. 3 **a** The circle with *Versus*. **b** The two periodic functions with *Line*

are at work in the *Versus* mode in which they are working. This is the mode where the time variable is implicit, and the two coordinated movements must fuse to make the figure, in this case either a rectangle, rhombus or circle. The diagram of these figures, belonging to the plane *ba*, is constituted of pairs (*b*, *a*), which capture specific positions in the plane and corresponding specific locations of the Wii remotes. Each position depends on the distances of the two controllers from the sensor during movement. If (b_1, a_1) and (b_2, a_2) are two distinct points of the diagram, they differ from each other in terms of movement, for the times at which the controllers reach the corresponding distances, say t_1 and t_2. So even if time seems to disappear in the *Versus* modality, it is obviously crucial for the creation of the diagrams, since these depend on movement. Indeed, what marks this modality as crucial for the purposes of this chapter is that the movements of the two users need to be coordinated to obtain the specific shape.

For example, in the case of a square or a rectangle (with sides parallel to the Cartesian axes), each side is created by one controller staying still and the other moving at a constant speed. In the case of the horizontal sides, the user at distance *b* (horizontal coordinate) is the only one moving, while a vertical side requires just the user at distance *a* to move. Thus, instant by instant, speed is not the same for the two movements: while the one user keeps null speed, the speed of the other is to be different from zero. If we take another quadrilateral (like a non-squared rhombus or a slanted rectangle), the two movements imply again two different instantaneous speeds, but there is also the fact that the ratio between speeds is always constant. Finally, when the expected diagram is a circle, the coordination between the controllers is much more difficult, and introduces challenging ideas about the relationship of movement to resultant diagram. In the case of the circle, the need to produce a curved line makes the task different from all the previous ones that the students have engaged in. With the circle, the speeds still need to be different from each other, but the users have to move in varying speeds in the same unit of time: while one user is at maximum speed, the other is at minimum speed while having to change direction, and when one is accelerating, the other needs to decelerate, and vice versa. Most importantly, the ratio between the speeds now is not constant, requiring the users to modulate their accelerations. The two movements must be coordinated and in "agreement" insofar as they together form the desired figure.

The combined effect of the students' coordinated movement, assembling the two linear movements (one the horizontal variable, and the other the vertical variable) produces a non-linear movement—that being the circular movement that appears in the resultant 'real-time' graph of the circle. Each student's hand movement has its own rhythmic pattern, each acts as either the horizontal variable or the vertical variable, and in this case the hands together form a new body or assemblage, a third movement. Their speeds must be different but coordinated. In so doing, their two different movements become conjoined, producing a combined effect. The combined effect is produced through the shared timing of their movements, creating a new movement that assembles the circular, creating the non-linear out of the combinations of linear movements. Each hand movement has its own rhythmic pattern, and each hand must move at a different speed, and indeed at related rates of changed speed, in order to achieve the effect. Thus the two bodies are moving together but apart, and the coupling of these movements forms a third movement that belongs to neither of the original bodies.

This task involves the combined movement of the two girls and the WiiGraph diagram. The two girls look at the screen where the graph appears but also down at their two hands as they move back and forth. We see here how time is the medium by which these two different movements commingle, and that a shared time must be adopted in order to create a new curve. This shared synchronized time introduces a new dimension to the combined movement, and yet each of the two human bodies has to follow different rhythms and speeds to achieve the effect. Thus the task points to the fundamental role of time or duration in theories of embodied mathematics, underscoring the way that bodies are assemblages of speeds and movements, while problematizing how we typically understand a body. We see how there is bodily agreement or coordination that characterizes the process of assembling, an aspect of group formation or body formation that is often overlooked. Agreement, as we use it, does not mean identification amongst parts, nor the creation of a unified homogeneous assemblage, but is used here to describe the coordination of heterogeneous movements – for instance a symbiotic relationship entails an agreement between two very different bodies that move together in a productive assemblage without erasing their distinctness. It is not that the two girls form bonds because they identify with each other, but because they are to become *coordinated* together. In that sense, there is a strong spatial element involved in the affective bonding that we see in forming this learning assemblage.

In the final part of the activity under discussion, after achieving the circle in the *Versus* modality (Fig. 3a), the researchers changed the software from *Versus* to *Line* modality. This modality calls time back as the independent variable in the space-time graphs, which capture over time the movement of the two Wiimotes as the changing distance from the sensor. The same two girls worked the controllers, and were asked to continue the movements that produced the circle in the other modality. The researcher repeated again and again "continue, continue" but said nothing else. The technology does not allow for having both space-time graphs and the circle (the spatial graph) present at the same time on the screen. So, the students have now to re-assemble in imagination the new coordinated movement of the circle graph, but now they produce the graphs of two periodic functions. The periodic functions that appear on the screen are the effect of using the *Line* modality (see Fig. 3b). The periodic functions (one for each girl) repeat horizontal or vertical coordinates according to the winding of the circular movement, graphed against the time variable. The class now sees the two girls continue to make the same hand movements, but now instead of a circle they generate two sinusoidal graphs. The teaching experiment helps the students grasp the many different ways in which related movements are at work in the apparently fixed and familiar figure of the circle, deepening their understanding of the geometric figure, and enhancing their embodied understanding of the mathematics involved. The *Line* modality shows the wave function for each of these movements, and shows where they intersect, directing attention to when the movements must be in some sense 'equal'.

In unexpected ways, this series of activities has "closed" the circle. From the initial experiences, which involved the researcher's hand as a catalyst of rhythm and speed, to the coordinated movements that produced the circle graph, learners pass to this last motion experience, which makes present the rhythm and speed of time in the production of the circle. In other words, moving back and forth between the *Line* and *Versus* modality allows the students to see how the same movements generate a circle and two periodic functions, thereby bringing to agreement these two ways of thinking about the movements inherent in making shapes. The periodic graphs demonstrate and indeed emphasize *the coordinated rhythm* of the students' movements.

5 Discussion: Assemblage Theory

In this section, we use *assemblage* theory to analyse the kind of mathematics learning that occurs as students assemble with technology in mathematics classrooms. The experiment discussed in the previous section sheds light on *innovative* technologies for teaching and learning mathematics. Assemblage theory, however, helps us analyse this data less in terms of tool use and more in terms of the affective force of the technology, insofar as it participates in the learning assemblage. The focus on learning assemblages is new *per sé* in the panorama of mathematics education literature on technology, which has tended to study technology in terms of 'tool use' and the affordances *for* the human. Here we look beyond the human at the entire assemblage that incorporates various kinds of non-human agencies. It is important to note, however, that an assemblage is *not* merely a set of bodies collected as one. Thus, it is not simply a matter of a student and a computer being seen as a cyborg, as for example recognized by Borba and Villareal (2005) when they speak of knowledge production by collectives of "humans-with-media". These collectives are "the basis for an epistemology that focuses attention on how people know things in different ways with the introduction of different technologies." (p. 27). Although our approach shares much with endeavour, there is an important difference regarding the episto-ontological claims. These authors state that the human should be considered as the epistemological subject, as the basic unit for thinking and of analysis in the production of knowledge. They focus on knowledge production and retain a dialectical relationship between the human and the technology: "We claim that a new technology of intelligence results in a new collective that produces new knowledge, which is qualitatively different from the knowledge produced by other collectives." (p. 24). We do not want to abandon this insight, but we want to draw attention to the more than human ontological relationships entailed in such a collective. Our chapter aims to study human-technology interaction without treating the human as a 'user' of a 'tool', because such an approach tends to over-emphasize human will and agency, as seen in the French perspective of instrumental genesis (e.g. Artigue, 2002; Baron, Guin, & Trouche, 2007). In other words, we aim to rethink the nature of distributed agency across an assemblage, and this entails rethinking the very nature of "use-value" since the idea harbours particular assumptions about agency (see for example the discussion around instrumental orchestration presented in Thomas et al.'s contribution in the book—Chap. Innovative Uses of Digital Technology in Undergraduate Mathematics). Moreover, we want to explore how the assemblage is assembled, rather than start with pre-given ideas about an individual who mediates outside sources of knowledge, like in the case of semiotic mediation or representational infrastructure theories (see Bartolini Bussi & Mariotti, 2008; Hegedus & Moreno-Armella, 2008). As Chorney (2014) points out, all too often "when the focus is on the student and the tool interacting, a dualist approach has been adopted." (p. 60). So, assemblage theory gives us a new perspective that helps us study the *more than human* process of "becoming-together", whereby the Wii, the students and the circle are entangled in the relational movement that characterises mathematical activity.

In particular, assemblage theory furnishes innovative ways of analysing the classroom episode, focusing on the affective and ethical nature of material entanglement. In the episode presented, the learning assemblage and its movement gradually emerge through various coordinated body movements, first between Barbara and Lucrezia, who must move so as to *agree* with the circular trajectory actualized by the researcher. The coordinated movement of the task re-assembles the students' hands into one movement forming a new body, that which actualizes the sinusoidal functions on the screen. The assembling process entails an ongoing agreement amongst various movements, which allows the collaborative activity to achieve something: that being a series of graphs that express mathematical relationships. We see in this coordinated agreement a way of addressing the ethical aspects of such activities, in that the students gradually and quietly become sympathetic with each other. This sympathy or agreement amongst various movements entails an ethical obligation to get the task done, or, following Barad (2010), shows us how "entanglements are relations of obligation—being bound to the other—enfolded traces of othering" (p. 265). The fact that bodies are related in terms of coordinated movements, and the fact that a graph is produced, relates directly to the force of affect that sustains the assemblage. Affect is the force or glue that sustains the sympathetic relations of agreement between Barbara and Lucrezia. Although it is difficult to track the evidence of this force, doing so helps us think about the ethical dimension of learning assemblages—there is an ethical obligation to the assemblage and its movement because affect glues the assemblage together (provisionally). There is a certain responsibility, which is part of the sympathetic manner in which the two girls are working together. If we think of this as a positive learning encounter, then this obligation to collaboratively engage with the task depends on the force of affect—engaging in the task depends on the fact that bodies have a capacity to be affected. Obligation is then a kind of coordinating with the other, a sympathetic agreeing with the other—not an identification, but a coordinated effort. The students affect each other. It is the power to be affected and to cause affects that produces the learning assemblage. For Deleuze, assembling *is* sympathy.

DeLanda (2006, 2011) will speak of assemblages as emergent entities within systems of matter, energy and information. The simplest assemblage, according to DeLanda, is formed when two molecular populations of air (or water) at different pressure or temperature are placed in contact. Because of the difference, a gradient is formed. This gradient is the simplest assemblage, having a tendency to dissipate but also a capacity to be exercised. Note that DeLanda (2011) defines assemblages in terms of a mathematical concept—the gradient—which is the derivative of a multi-variable function. A gradient is a vector whose components are the partial derivatives of an n-dimensional function. In this way, DeLanda operationalizes and makes more concrete the proposal that assemblages are relations of speed and movement. He is not simply suggesting that gradient is a good metaphor for how complex assemblages are formed. He is literally suggesting that assemblages are differentiation processes and relations of difference. As in our case study, the assembling of girls and technology is a gradient (a series of relations of speed and direction). Quite explicitly, the technology entails that Lucrezia and Barbara's instantaneous speeds are captured as the two derivatives $\mathrm{d}b/\mathrm{d}t$ and $\mathrm{d}a/\mathrm{d}t$ that constitute the speed of movement along the circle, as well as that their directions make the direction of this movement. Briefly speaking, WiiGraph assembles the derivatives in the slope of the line tangent to the circle point by point. This case study with the circle illustrates DeLanda's argument about how *assemblages are gradients*—quite literally, the study directs our attention to how the concept of circle is a series of speeds coordinated through the students' movements. Their timed accelerated movements *are* the gradients that are imperceptible in the graph. The assemblage of graph-concept-student is achieved through these gradients.

Our case study illustrates this point well because the graphs on the screen are produced only by a series of coordinated movements, and thus the achievement of a circle graph is in fact nothing more than a complex set of differentials (degrees of difference). What we take to be the unity of the achievement is actually a field of movement and differential relations of speeds and directions. All of these components partake of the mathematical work in the classroom; the mathematical work is determined by the relations between the components. For example, when the hand movements of two students are assembled with each other, an entirely new kind of movement emerges—a circular and also periodic movement emerges from the combining of two linear movements. Thus mathematical concepts of linearity, periodicity, etc. are at play in the emergence of this particular assemblage. The speeds of the movements are intrinsically relevant to the resultant properties of this assemblage—that being the circle graph and later the sinusoidal graphs. The circle emerges as a specific varying relationship of speed and direction, and the sinusoidal as a specific relation of varying slopes, both graphs emerging from the modulation of Lucrezia and Barbara's accelerations. The case study demonstrates how the radically new—in this case circular movement—can emerge from a set of components that are different in kind (linear). The linear can be combined to create the circular. We often take this for granted, but it is actually a philosophically significant action. The fact that the circular motion emerges in this way is a perfect example of how an emergent property can be distinctive and not possessed by the components of the assemblage.

It is important to understand that the identity of an assemblage is both embodied and *expressed* in its materiality. In other words, an assemblage is associated with a body (an individualized collective) *and* expressed information as "raw physical pattern." (DeLanda, 2011, p. 200). This dual emphasis on embodiment and expression is crucial for any theory of assemblage that aims to attend to learning. A learning assemblage must thrive through its gradient while also *express information*. A gradient in physics can be live or dissipated, and in either case it possesses the same energy. However, the live gradient expresses more information because it is ordered. A dissipated gradient lacks order. A high degree of information is associated with a highly structured assemblage. We can see in our case study how the assemblage of students and Wii technology is highly structured insofar as the students have been asked to do something and they are attempting to do it. Like any classroom, there is some authority that structures the activity. But the assemblage is also highly structured in a more systems-based way insofar as the various movements of components—the two different students, the software, the sensor—are carefully coordinated so that they can create the desired graph on the screen. For example, Lucrezia and Barbara cannot see any mark on the screen if they do not keep the controllers pointed at the sensor. Coordination between the students' movements and the sensor requires such a careful pointing during the motion experience. In a similar way, the girls need the software feedback to grasp the efficiency of their movements. They even do not know a priori that their movements need to respect the conditions discussed above. It is only when their coordinated movement realizes and sustains these conditions that they are able to assemble with the technology in a sympathetic way that they are then able to draw a circle-like diagram. It is precisely at this point that they experience the shared obligation to each other that Barad cites above, that sense of shared commitment and affective bond that achieves the coordinated rhythms and the circle graph.

The repeated attempts of the girls to achieve the graph shows how sympathies proliferate in everyday minute interactions, lived in and as affective bonds, and assemble into larger overt coordinated emotional responses between bodies. Minute sympathetic movements contribute to passionate attachments, so that the emotional investment in such shared activities becomes pronounced: "sympathy is bodies who love or hate each other, each time with populations in play, in these bodies or on these bodies." (Deleuze & Parnet, 2007, p. 52). Thus affect circulates across minute movements as the two girls coordinate their activity. We see the learning assemblage evolve through these relations, where sympathy becomes "something to be reckoned with, a bodily struggle". The girls do not identify with each other or 'put oneself in the other's shoes', but they assemble with each other and with the Wii, and thereby enter a process of *becoming other* that does not erase the other (Deleuze & Parnet, 2007, p. 53). An ethical relationship emerges through the sympathetic coordination of movement.

Thus there is an increasingly intense obligation amongst the many components of the assemblage as the teaching experiment unfolds. The assemblage is embodied in the relations between these participants, but is also expressed through *and in* the information in the graph. That information is precisely the mathematical relationships captured in the graph. There are many different ways in which the students might have made some marks on the screen using the WiiGraph technology, and all these different ways express different degrees of information. The desired outcome —the circle—is clearly considered as that which possesses more information by the adults present. Thus the sense of obligation that an entanglement entails, this sense that we are entangled together in the shared task, is both an affective sympathetic bond and an expression of information deemed to be information by the researchers who are present. DeLanda is careful to situate assemblages in historical and cultural context, and to recognize the contingency of how particular arrangements and movements are deemed to possess more information than others. Although we do not have the space to develop these ideas here, his approach is not a neutral or ahistorical theory of assemblages, but rather one that brings together insights from systems theory (in particular physical-chemical processes at work in systems) with affect theory and information theory, in such a way that the historical-political context is also integrated.

Summarizing, in this chapter we have used assemblage theory to offer new ways of examining the processes through which individual human bodies come together with technology in mathematical activity. Our innovative analysis highlights the *relational movement* that characterises the *entanglement* of the technology, the students and the mathematical concepts. The emphasis on relations between movements has allowed us to study how the entire learning assemblage is doing mathematics and thinking mathematically. With our episode we have proposed to look at the process of becoming-together as more than human, instead of focusing on the human as a user of tools. We have also seen how we have been able to frame the discussion of the classroom episode shedding light on the degree of obligation that is entailed in any assemblage, and how this helps us begin to think about the ethical dimension of learning, where questions of obligation and responsibility must be considered. This has brought us to draw attention to the role of affect in innovative technology and to think of it as the force that sustains a sympathetic human-technology assemblage. Affect is intended in Deleuzian terms, as the force that nourishes sympathetic relations of agreement between bodies whereby they are mutually affected by each other. Affect thus circulates across bodily relations of sympathy. Sympathy takes on a pivotal role in any learning assemblage. Accordingly, it is not a matter of identification but of coordinated effort of *agreeing* with the other—this agreeing allows for radically diverse forms of heterogeneous movement, and is not a matter of compliance or becoming the same. It is rather an attempt to think about how we form assemblages of radically heterogeneous movements in ways that are productive of learning and ethical relationships. In our case study, the bodily agreement or coordination produces rich mathematical thinking—an assembling of gradients and directions that speaks directly to the shape of the sinusoidal functions and their relationship with the figure of the circle. The learning assemblage that we have analysed here is a complex entanglement of affect and information, demonstrating how innovative technologies add to our understanding of fundamental aspects of mathematics learning.

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